Chapter 6
Analysis of Time Series

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6.1. Introduction

Intelligent data analysis often requires one to extract meaningful conclusions about a complicated system using time-series data from a single sensor. If the system is linear, a wealth of well-established, powerful techniques is available to the analyst. If it is not, the problem is much harder and one must resort to nonlinear dynamics theory in order to infer useful information from the data. Either way, the problem is often complicated by a simultaneous overabundance and lack of data: megabytes of time-series data about the voltage output of a power substation, for instance, but no information about other important quantities, such as the temperatures inside the transformers. Data-mining techniques [177] provide some useful ways to deal successfully with the sheer volume of information that constitutes one part of this problem. The second part of the problem is much harder. If the target system is highly complex—say, an electromechanical device whose *dynamics* is governed by three metal blocks, two springs, a pulley, several magnets, and a battery—but only one of its important properties (e.g., the position of one of the masses) is sensor-accessible, the data analysis procedure would appear to be fundamentally limited.

Fig. 6.1 shows a simple example of the kind of problem that this chapter addresses: a mechanical spring/mass system and two time-series data sets gathered by sensors that measure the position and velocity of the mass. This system is linear: it responds *in proportion to* changes. Pulling the mass twice as far down, for instance, will elicit an oscillation that is twice as large, not one that is $2^{1.5}$ as large or $\log 2$ times as large. A pendulum, in contrast, reacts *nonlinearly*: if it is hanging straight down, a small change in its angle will have little effect, but if it is balanced at the inverted point, small changes have large effects. This

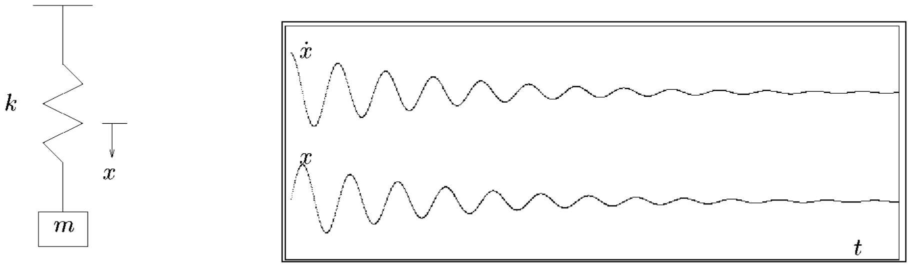

Fig. 6.1. A simple example: A spring/mass system and a time series of the vertical position and velocity of the mass, measured by two sensors

distinction is extremely important to science in general and data analysis in particular. If the system under examination is linear, data analysis is comparatively straightforward and the tools—the topic of section 6.2 of this chapter—are well developed. One can characterize the data using statistics (mean, standard deviation, etc.), fit curves to them (functional approximation), and plot various kinds of graphs to aid one's understanding of the behavior. If a more-detailed analysis is required, one typically represents the system in an "input + transfer function → output" manner using any of a wide variety of time- or frequency-domain models. This kind of formalism admits a large collection of powerful reasoning techniques, such as superposition and the notion of transforming back and forth between the time and frequency domains. The latter is particularly powerful, as many signal processing operations are much easier in one domain than the other.

Nonlinear systems pose an important challenge to intelligent data analysis. Not only are they ubiquitous in science and engineering, but their mathematics is also vastly harder, and many standard time-series analysis techniques simply do not apply to nonlinear problems. Chaotic systems, for instance, exhibit broad-band behavior, which makes many traditional signal processing operations useless. One cannot decompose chaotic problems in the standard "input + transfer function → output" manner, nor can one simply low-pass filter the data to remove noise, as the high-frequency components are essential elements of the signal. The concept of a discrete set of spectral components does not make sense in many nonlinear problems, so using transforms to move between time and frequency domains—a standard technique that lets one transform differential equations into algebraic ones and vice versa, making the former much easier to work with—does not work. For these and related reasons, nonlinear dynamicists eschew most forms of spectral analysis. Because they are soundly based in nonlinear dynamics theory and rest firmly on the formal definition of invariants, however, the analysis methods described in section 6.3 of this chapter do not suffer from the kinds of limitations that apply to traditional linear analysis methods.