Chapter 9
Fuzzy Logic

Michael R. Berthold
Data Analysis Research Lab, Tripos Inc., USA

9.1. Introduction

In the previous chapters a number of different methodologies for the analysis of datasets have been discussed. Most of the approaches presented, however, assume precise data. That is, they assume that we deal with exact measurements. But in most, if not all real-world scenarios, we will never have a precise measurement. There is always going to be a degree of uncertainty. Even if we are able to measure a temperature of 32.42 degrees with two significant numbers, we will never know the exact temperature. The only thing we can really say is that a measurement is somewhere in a certain range, in this case $(32.41, 32.43)$ degrees. In effect, all recorded data are really intervals, with a width depending on the accuracy of the measurement. It is important to stress that this is different from probability, where we deal with the likelihood that a certain crisp measurement is being obtained [558]. In the context of uncertainty we are interested in the range into which our measurement falls. Several approaches to handle information about uncertainty have already been proposed, for example interval arithmetic allows us to deal and compute with intervals rather than crisp numbers [388], and also numerical analysis offers ways to propagate errors along with the normal computation [34].

This chapter will concentrate on presenting an approach to deal with imprecise concepts based on *fuzzy logic*. This type of logic enables us to handle uncertainty in a very intuitive and natural manner. In addition to making it possible to formalize imprecise numbers, it also enables us to do arithmetic using such *fuzzy numbers*. Classical set theory can be extended to handle partial memberships, thus making it possible to express vague human concepts using *fuzzy sets* and also describe the corresponding inference systems based on *fuzzy rules*.

Another intriguing feature of using fuzzy systems is the ability to granulate information. Using fuzzy clusters of similarity we can hide unwanted or useless information, ultimately leading to systems where the granulation can be used to focus the analysis on aspects of interest to the user.

The chapter will start out by explaining the basic ideas behind fuzzy logic and fuzzy sets, followed by a brief discussion of fuzzy numbers. We will then concentrate on fuzzy rules and how we can generate sets of fuzzy rules from data. We will close with a discussion of Fuzzy Information Theory, linking this chapter to Appendix B by showing how Fuzzy Decision Trees can be constructed.

9.2. Basics of Fuzzy Sets and Fuzzy Logic

Before introducing the concept of fuzzy sets it is beneficial to recall classical sets using a slightly different point of view. Consider for example the set of "young people", assuming that our perception of a young person is someone with an age of no more than 20 years:

$$\text{young} = \{x \in P \mid \text{age}(x) \leq 20\}$$

over some domain P of all people and using a function *age* that returns the age of some person $x \in P$ in years. We can also define a characteristic function:

$$m_{\text{young}}(x) = \begin{cases} 1 & : \quad \text{age}(x) \leq 20 \\ 0 & : \quad \text{age}(x) > 20 \end{cases}$$

which assigns to elements of P a value of 1 whenever this element belongs to the set of young people, and 0 otherwise. This characteristic function can be seen as a *membership function* for our set *young*, defining the set *young* on P.

Someone could then argue with us that he, being just barely over 20 years old, still considers himself young to a very high degree. Defining our set *young* using such a sharp boundary seems therefore not very appropriate. The fundamental idea behind fuzzy set theory is now a variable notion of membership; that is, elements can belong to sets to a certain degree. For our example we could then specify that a person with an age of, let's say, 21 years, still belongs to the set of *young* people, but only to a degree of less than one, maybe 0.9. The corresponding membership function would look slightly different:

$$\mu_{\text{young}}(x) = \begin{cases} 1 & : \quad \text{age}(x) \leq 20 \\ 1 - \frac{(\text{age}(x)-20)}{10} & : \quad 20 < \text{age}(x) \leq 30 \\ 0 & : \quad 30 < \text{age}(x) \end{cases}$$

Now our set *young* contains people with ages between 20 and 30 with a linearly decreasing degree of membership, that is, the closer someone's age approaches 30, the closer his degree of membership to the set of young people approaches zero (see Figure 9.1).